

Math 251 Midterm 1 Sample

Name: _____

This exam has 10 questions, for a total of 100 points.

Please answer each question in the space provided. You need to write **full solutions**. Answers without justification will not be graded. Cross out anything the grader should ignore and circle or box the final answer.

Question	Points	Score
1	10	
2	15	
3	10	
4	10	
5	10	
6	10	
7	5	
8	10	
9	10	
10	10	
Total:	100	

Question 1. (10 pts)

In the following, \mathbf{a} , \mathbf{b} and \mathbf{c} are nonzero vectors in \mathbb{R}^3 .

(a) Does the expression $\mathbf{a} \times (\mathbf{b} \bullet \mathbf{c})$ make sense?

(b) If $\mathbf{a} \times \mathbf{b} = \mathbf{0}$, what is the angle between \mathbf{a} and \mathbf{b} ? List all possibilities.

(c) Given the surface $z^2 + x^2 - y^2 - 6z + 2x + 6 = 0$, determine its type:

- A. ellipsoid
- B. elliptic paraboloid
- C. cone
- D. hyperboloid of one sheet
- E. hyperboloid of two sheets

Question 2. (15 pts)

- (a) Find an equation of the plane that passes through the point $(3, 3, 1)$ and is orthogonal to the line

$$x = t, \quad y = 2 + t, \quad z = 3t.$$

- (b) Find the angle between the plane in part (a) with the plane $x - y - 3z = 1$

- (c) Find the line of intersection of the plane in part (a) with the plane $x - y - 3z = 1$.

Question 3. (10 pts)

A curve is described by the vector function $\mathbf{r}(t) = \langle \sin \pi t, \sqrt{t}, \cos \pi t \rangle$.

(a) Find the derivative of $\mathbf{r}(t)$.

(b) Find the tangent line to this curve at the point $(0, 1, -1)$.

Question 4. (10 pts)

Determine whether the following limit exists or not. Show work!

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$$

Question 5. (10 pts)

Let $z = 5x^2y + y$ with $x = s \cos t$ and $y = s^2 + e^t$. Find the value of $\frac{\partial z}{\partial t}$ for $(s, t) = (1, 0)$.

Question 6. (10 pts)

A surface is given by an equation

$$x^2 + y^2 - 2z^2 + xyz = 2$$

Find the tangent plane of this surface at the point $(0, 2, 1)$

Question 7. (5 pts)

Find all second partial derivatives of the function $f(x, y) = e^{x^2 - y^2}$.

Question 8. (10 pts)

Given the equation $xe^z = y^2 \sin(xyz) + 1000$, find $\partial z / \partial y$ by using implicit differentiation.

Question 9. (10 pts)

Given the function

$$z = \sqrt{y^2 - x}$$

(a) Find the gradient of the function

(b) Find the maximum rate of change of the function at the point $(5, 3)$, and determine in which direction this maximum occurs.

Question 10. (10 pts)

Use differentials to approximate the number $\sqrt{3.96} \ln(1.07)$.